



Loza, AT., Canagarajah, CN., & Bull, DR. (2003). Adaptive SPWV distribution with adjustable volume 2-D separable kernel. In *10th IEEE International Conference on Electronics, Circuits and Systems, 2003 (ICECS 2003)* (Vol. 1, pp. 68 - 71). Institute of Electrical and Electronics Engineers (IEEE).
<https://doi.org/10.1109/ICECS.2003.1301978>

Peer reviewed version

Link to published version (if available):
[10.1109/ICECS.2003.1301978](https://doi.org/10.1109/ICECS.2003.1301978)

[Link to publication record in Explore Bristol Research](#)
PDF-document

University of Bristol - Explore Bristol Research

General rights

This document is made available in accordance with publisher policies. Please cite only the published version using the reference above. Full terms of use are available:
<http://www.bristol.ac.uk/red/research-policy/pure/user-guides/ebr-terms/>

ADAPTIVE SPWV DISTRIBUTION WITH ADJUSTABLE VOLUME 2-D SEPARABLE KERNEL

Artur Łoza, Nishan Canagarajah and Dave Bull

University of Bristol, Centre for Communications Research
Merchant Ventures Building, Woodland Road, Bristol BS8 1UB, UK

ABSTRACT

In practical Time-Frequency (T-F) analysis of nonstationary signals associated with the use of the Smooth Pseudo Wigner-Ville distribution (SPWV) an important issue is a choice of the smoothing window. To address this problem, a fast sub-optimal technique is proposed, relating the window length to the approximate slope of the frequency modulation. The results obtained by using this automated procedure are comparable to results given by supervised decompositions and are more (locally) adapted. The developed method has been used in processing of radar returns. In order to objectively assess the performance of the transforms several measures can be used. For signals with unknown modulation laws cost functions are available, including measures based on entropy function. Their application to evaluation of the obtained T-F distributions is demonstrated.

1. INTRODUCTION

Most of the Time-Frequency Representations (TFRs) employ a smoothing kernel or a window to reduce cross-components. The choice of the smoothing window length significantly affects quality of the resulting T-F image. The smoothing parameters of the TFR need to be chosen carefully, based on the visual investigation or *a priori* knowledge of the signal contents. If the signal contains different components, it may be impossible to choose one value of the parameter, suitable for the whole series. This calls for a time adaptive approach. The purpose of the developed adaptive schema is to automatically choose an appropriate window (or at least sub-optimal one), matched locally to the signal components, based on local 1-D time and frequency moments of the signal. Unlike other signal adaptive representations (see [1], [2] for example) that operate in 2-D domain of T-F distribution, our method chooses window width prior to computation of the SPWV. The technique does not guarantee optimality but can be used to produce an unsupervised sub-optimal smoothing kernel.

2. BACKGROUND AND DEFINITIONS

Cohen Class distributions can be written as the double convolution of the Wigner Ville distribution (WV) of the sig-

nal and a two dimensional T-F smoothing function L [3],

$$TFR_x(t, \omega; L) = WV_x(t, \omega) * L(t, \omega),$$

where WV_x , is defined as

$$WV_x(t, \omega) = \int x(t + \frac{t'}{2})x^*(t - \frac{t'}{2})e^{-i\omega t'} dt'.$$

Different TFRs can be obtained from the fundamental WV distribution by applying a different smoothing function L . In our experiments a Gaussian separable 2-D function has been used, with the time and frequency widths as parameters. A Gaussian function is used for its optimum T-F localisation property [3]. Here, instead of defining each width separately, we first choose one of the lengths, α_0 , and adjust the volume of the kernel with parameter v , in the spirit of coupled smoothing,

$$L(t, \omega) = g(t)H(\omega) = 1/v^2 \exp(-t^2/\alpha^2 - \omega^2/\beta^2),$$

where $\alpha = v\alpha_0$, and $\beta = v/\alpha_0$. The positive distributions are obtained for $v \geq 1$ [3]. When $v = 1$ is used, the smoothed distribution is equivalent to the Spectrogram (SP). In such cases, L is the WV distribution of the SP smoothing window of width $\sqrt{2}\alpha_0$.

As shown in [3], for a chirp with a slope c , $x(t) = Ae^{ic(\tau-t)^2/2}$, the frequency spread of the spectrogram with smoothing function $h = e^{-t^2/\alpha^2}$, attains a minimum for $\alpha_{opt} = 1/\sqrt{\pi|c|}$. In our method we relax the optimality constraint and approximate the slope of a modulation c of arbitrary signal with a diagonal of the TB rectangular, computed on the segment of the signal: $c = B_x/T_x$. This idea is illustrated in Figure 1 (a). The developed algorithm operates in 1-D to estimate moments of the signal in time and frequency domains. Considering $|x(t)|^2$ and $|X(\nu)|^2$ as probability distributions we can compute moment estimates [3] (limits of integration are $-\infty, +\infty$):

$$\begin{aligned} T^2 &= \frac{4\pi}{E_x} \int (t - t_m) |x(t)|^2 dt, \\ B^2 &= \frac{4\pi}{E_x} \int (\nu - \nu_m) |X(\nu)|^2 d\nu \end{aligned} \quad (1)$$

where $E_x = \int |x(t)|^2 dt < +\infty$ is the bounded signal energy and t_m and ν_m are mean time and frequency positions, respectively. Throughout this paper, k and l will stand for appropriately discretised time and frequency coordinates.

This work is supported by AMS UK.

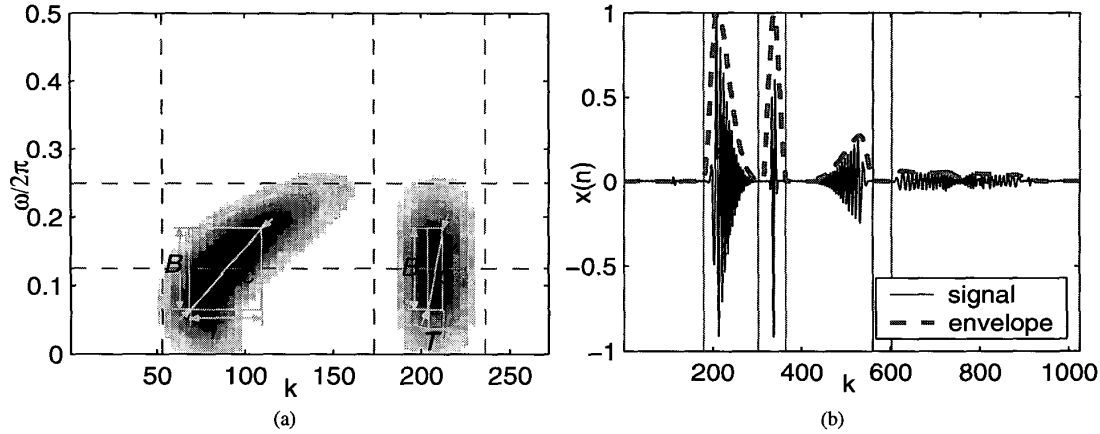


Figure 1. Approximating the slope c of the frequency modulation with use of time and frequency moments, T and B respectively. Graph (a): the plot of the $T \times B$ rectangle estimated on the 1-D data is superimposed on the T-F image of the signal. The time and frequency intervals used in the moments estimation are marked with red lines. Graph (b) depicts an example of the adaptive segmentation of the signal.

3. ADAPTIVE TFR

The whole TFR computation procedure can be divided into three main stages:

I. Segmentation of the signal: The data is divided into segments of variable length, depending on the signal structure. Specifically, the segments are determined based on the local minima of the signal envelope. The envelope A_x reflecting the shape of the instantaneous amplitude of the signal x is computed by computing local maxima,

$$x_{max}(k) = \max_j [x(k-j)],$$

and then averaging them,

$$A_x(k) = \frac{1}{M} \sum_j x_{max}(k-j).$$

In both steps, signal portions of M samples are taken, centred around a sample k , and hence, in the equations above, $j = -M/2 + 1, \dots, M/2$, for even values of M . Choice of M determines the smoothness, or 'sensitivity' of the envelope and can be determined experimentally. Minima localized close to each other or corresponding to small fluctuations of the envelope are discarded. This segmentation also allows detection of negligible energy intervals that can be excluded from further analysis to reduce computation load.

II. Finding the suitable window lengths: Moments are computed in subbands of the frequency plane of a segment of the signal. Subbands are obtained by dividing the frequency axis into two or four parts. Overlapping frequency moments are merged and non-overlapping ones are energy-weighted (e.g. $B = (E_1 B_1 + E_2 B_2)/(E_1 + E_2)$ where E_n and B_n correspond to energy and width of n -th subband) and then a final estimate for the segment is computed. Values of time and frequency moments of the

signal were converted to window lengths using relations shown in Section 2.

III. Computing of the TFR: In order to avoid border distortions caused by signal segmentation, overlap between segments is introduced. The TFR decomposition of each segment is then computed with the use of calculated parameters and the central part of the output (i.e. after discarding the overlap) is used.

The developed schema was applied to the SP and the SPWV decompositions by varying parameter v in smoothing function. Extension to other types of windows is possible, after relation between window time spread and the window length is established.

4. RESULTS

For presentation purposes, a signal with a high frequency modulation rate of components was chosen. The signal with a high modulation rate was considered to be difficult to analyse because of overlapping transients and frequency modulated signals. The standard method used in the comparison is a decomposition performed on the whole signal with one window chosen by a visual inspection. Additionally, the Adaptive Optimal Kernel (AOK) [1] procedure was used with parameters: analysing window of length 64 samples, kernel volume parameter 1.5. The results are shown in Figure 2.

By comparing the plots it can be concluded that adapting smoothing window lengths in some TFRs yields improved results compared to supervised experiments. The ridges of the TFR image becomes thinner without increasing the level of cross-components. The algorithm also distinguishes between radically different components such as impulses and chirps within one signal. Although the AOK performed well, there seem to be some artifacts and loss of information when using this distribution.

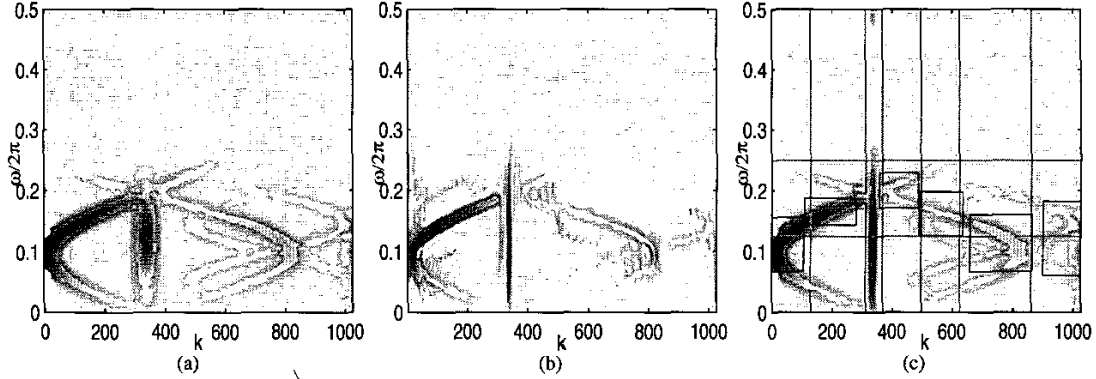


Figure 2. The comparison of standard and adaptive SPWV decompositions : (a) standard methods (b) the AOK, (c) the developed method. Plots superimposed on TFR image: squares show the window lengths; dashed vertical lines show segments boundaries.

5. ASSESSMENT OF THE TFRS

5.1. Information cost functions

The performance of the T-F distributions of signals with unknown properties can be assessed by using appropriate cost functions. Here, the evaluation will be performed with the use of simulated radar data. Several performance measures can be used in the field of the T-F analysis (see [4] for examples). Generally, signal independent non-parametric cost functions are desired. However, parametric measures, such as the number of samples above a threshold $N(\epsilon)$, can give reliable results, providing that an appropriate threshold can be estimated, based on the dynamic characteristic of the signal. In this study, following cost functions are considered:

Number of samples above the threshold [5],

$$N(\epsilon) = \sum_k \sum_l \varrho(TFR_x(k, l)),$$

where $\varrho(TFR_x(k, l)) = 1$ if $|TFR_x(k, l)| > \epsilon$. Threshold based on the maximum amplitude, $\epsilon = \epsilon \max(|TFR_x|)$, $\epsilon = 0.001$ was used.

Generalized entropy of Rènyi [4],

$$R_\alpha = \frac{1}{1 - \alpha} \log_2 \sum_k \sum_l TFR_x^\alpha(k, l).$$

It is a measure of signal complexity (proportional to a number of signal components). Rènyi entropy becomes equivalent to Shannon entropy as $\alpha \rightarrow 1$, but the negative values of most Cohen Class representations prohibit the use of this measure, due to the logarithm in the equation. However, for evaluation purposes, it was decided to only use the positive values of the distributions when computing Shannon entropy. For $\alpha > 1$, Rènyi measure, unlike Shannon entropy, is immune to negative values of the Time-Frequency Representation (TFR). Odd order Rènyi entropy (e.g. $\alpha = 3$) measures auto-component concentration and even order Rènyi entropy (e.g. $\alpha = 2$) measures cross-component suppression. Thus, by minimization the

differential entropy $R_{32\gamma} = R_3 - \gamma R_2$, $0 \leq \gamma \leq 1$, the distribution can be evaluated with respect to the trade-off between resolution and cross-components presence. In practice, the values of γ close to 1 are used, more appropriate for well-smoothed representations.

5.2. Evaluation examples

It is understood that the best performance measure is the one that is linked to desired features. In this study, we search for good T-F resolution and concentration, the ability to resolve close or overlapping components and low amount of cross-components. It is difficult to find a single function satisfying all of the above and thus three measures were used: Shannon entropy, differential Rènyi entropy and number of samples above the threshold.

Several well-performing TFRs were chosen for comparison: the WV, the SP, the Zao-Atlas-Marks distribution (ZAM) [6], the SPWV with $v = 0.75$ and $v = 0.5$, and the adaptive SPWV (referred to as "ASPWV"). For visualization purposes the output of the assessment procedure was normalized: cost values were scaled so that they are within the same range and they decrease for TFRs having good properties. Results are presented in Figure 3

Based on the plots it was concluded that most of the cost functions yield results that confirm visual evaluation of the images. However, for TFRs with very good properties and small differences the cost functions yield very similar values which make it difficult to compare them. Both Shannon entropy and differential Rènyi entropy measure concentration and are sensitive to cross-components; however, the influence of the concentration seems to be prevailing. This explains the low cost values of the under-smoothed SPWV in Figure 3. For most of the measurements, adaptive transform seems to produce lower cost values than the SP. Some degree of inconsistency is due to the complex structure of the multiple-component signals and the fact that the distribution properties are handled in a global way.

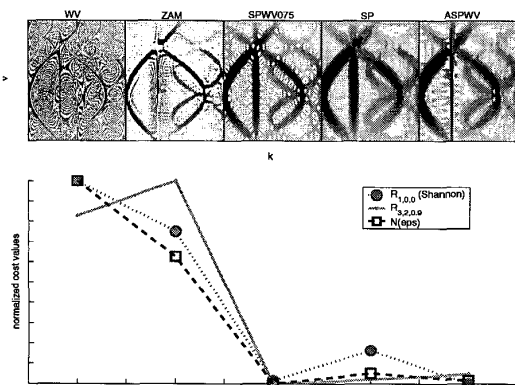


Figure 3. Assessment of performance TFRs of the test data.

6. CONCLUSIONS

An unsupervised and adaptive T-F decomposition procedure which requires minimal *a priori* knowledge of the signal has been developed. The technique is simple, fast and operates in 1-D time and frequency domains. The procedure proves to be applicable in a suboptimal way for signals containing both transients and frequency modulated signals. The results obtained are similar to those of conventional decompositions (the SP, the SPWV) and locally adapted.

The information cost functions, such as generalized entropy of Rènyi, can be used to assess the TFR of a multi-component signal. Practical experiments with use of complex, multiple-component signals, show that although generally consistent with the visual observations, the measures are subject to some variation, especially for the images of similar quality.

7. REFERENCES

- [1] Richard G. Baraniuk and Douglas L. Jones, "An adaptive optimal-kernel time-frequency representation," *IEEE Transactions on Signal Processing*, vol. 43, no. 10, pp. 2361–2371, 1995.
- [2] Douglas L. Jones and Richard G. Baraniuk, "A simple scheme for adapting time-frequency representations," *IEEE Transactions on Signal Processing*, vol. 42, no. 12, pp. 3530–3535, December 1994.
- [3] Patrick Flandrin, *Time-Frequency/Time-Scale Analysis*, Academic Press, 1999.
- [4] Richard G. Baraniuk, Patrick Flandrin, Augustus Janssen, and Olivier Michel, "Measuring time-frequency information content using the Rènyi entropies," *IEEE Transactions on Information Theory*, vol. 47, no. 4, pp. 1391–1409, May 2001.
- [5] Michel Misiti, Yves Misiti, Georges Oppenheim, and Jean-Michel Poggi, *Wavelet Toolbox For Use with Matlab; Users Guide*, The MathWorks, Inc., March 1996.
- [6] Yunxin Zhao, Les E. Atlas, and Robert T. Marks, "The use of cone-shaped kernels for generalized time-frequency representations of nonstationary signals," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. ASSP-37, no. 7, pp. 1084–1091, 1990.